3.4 The response of rent to changes in the tax rate depends on the supply and demand elasticities. The magnitude of the demand elasticity depends on whether or not the product is traded internationally. The magnitude of the supply elasticity, as we will see later on, depends on the length of run. For this reason, it is of interest to evaluate the expression $\hat{R}/\hat{T}$ for the extreme values of the supply and demand elasticities. Do it for the following:

(a) Let $0 < \epsilon_{ls} < \infty$ and evaluate $\hat{R}/\hat{T}$ for $\epsilon_{ld} \to 0$ and for $\epsilon_{ld} \to -\infty$.

(b) Let $0 > \epsilon_{ld} > -\infty$ and evaluate $\hat{R}/\hat{T}$ for $\epsilon_{ls} \to 0$ and for $\epsilon_{ls} \to \infty$

Interpret the results in terms of the impact on the price and output

3.5 Modify the case discussed in Figure 3.2 to an import ban, so that the country is forced to change its output from A to a point of no trade. Evaluate:

(a) The effect on p and real factor prices

(b) How does your answer to (a) depend on factor intensity?

(c) Indicate what tariff level would be required to achieve the same effect on the country.

3.6 Introduce land explicitly into the agricultural production function How do the following policies affect the rent on land:

(a) Subsidy to agriculture in (1) a closed economy, (2) an open economy.

(b) Tax on wages in (1) a closed economy, (2) an open economy.

(c) Food aid in (1) a closed economy, (2) an open economy.

3.7 Suppose all the rent was taxed away and distributed as a lump sum What effect would this tax have on the equilibrium position of the economy?

3.8 It has been argued that the elimination of protection of agriculture in the Common Market will improve world agricultural prices and thereby benefit less-developed countries (LDCs) that export food. Show how trade liberalization in a country that protects agriculture by imposing a tariff on imports will change the equilibrium position of this country and relate it to the foregoing argument

3.9 If trade liberalization is to be accepted as a norm of behavior for all countries, exporting countries will be required to eliminate taxes on exports What effect would this have on world agricultural prices?

4

The Consequences of Resource Changes

The output composition of the economy depends on resource endowment. Land-abundant countries are the main suppliers of cereals, and countries rich in labor and short on capital produce labor-intensive products. As such, differences in resource endowment explain cross-country differences in the composition of production. While the size of the country is fixed, its capital is not, and the growth process brings about changes in resources, thereby directly affecting the output composition. As we shall see in subsequent chapters, resources also affect the form and perhaps the pace of technical change, and thereby they affect the output composition indirectly

Continuing the discussion of Chapter 2, note that the initial resource supply is represented by k alone. The question then is what effect does a change in k have on the economy Because we are interested in growth, we concentrate on an increase in k, which we refer to as capital deepening Of course, the consequences of a decline in k, or labor deepening, can be evaluated in a similar way. The direct consequence of a change in k is on supply, and this is taken up first. This is followed by the analysis of the long-run equilibrium, where the term "long run" in this context indicates that resources are allowed to change. Such a terminology serves as an analytic convenience of isolating effects rather than as a description of reality. It is artificial to isolate resource change from technological change. The analysis is subsequently extended to allow resources to change in response to changes in economic variables. Under such varying factor supply the transformation curve is modified In the limiting case, when the supply of one of the factors is perfectly elastic, the transformation curve becomes a straight line, and the output composition is determined by demand. The size of the cultivated land is an economic variable The supply of land in any country is given by the boundaries of the country. Yet not all the land is cultivated, and furthermore, there are variations in the area under cultivation over time. The last part of the chapter introduces a framework for analyzing the variations of the area under cultivation.

The Model

We rewrite some key equations developed in Chapter 2:

$$\ell(\omega, k) = \frac{k_2(\omega) - k}{k_2(\omega) - k_1(\omega)},$$
$$y_1(\omega, k) = \ell(\omega, k)[f_1(k_1(\omega))],$$
$$y_2(\omega, k) = (1 - \ell(\omega, k))[f_2(k_2(\omega))],$$
$$x_1 = D(p, x_2)$$

We then have equilibrium in a closed economy:

$$z_i(p) = x_i(p) - y_i(p) = 0,$$

and equilibrium in a small open economy:

$$px_1(p) + x_2(p) = py_1(p) + y_2(p)$$

Supply

Output Response

The main result here is due to Rybczynski (1955).

PROPERTY 4.1 (supply; expansion-pattern) An increase in the capital-labor ratio, holding p constant, increases the output of the capital-intensive sector and decreases the output of the labor-intensive sector.

This result follows immediately from equation (2.13). Under constant prices, $k_i(\omega)$ are also constant, and hence $\partial\ell(\omega, k)/\partial k < 0$. With constant k_i, a decline in ℓ implies a decline in $\rho = K_1/K$. With prices constant, the only way for the economy to absorb a larger capital-labor ratio is to increase the share of the capital-intensive sector in total resources. Consequently, the total employment of resources in the labor-intensive sector declines and so does its output. The opposite is true for the capital-intensive sector. This is all summarized by taking partial derivatives of (2.14) with respect to k. For sector 1 we have

$$\frac{\partial y_1(\omega, k)}{\partial k} = f_1[k_1(\omega)]\frac{\partial\ell(\omega, k)}{\partial k}.$$

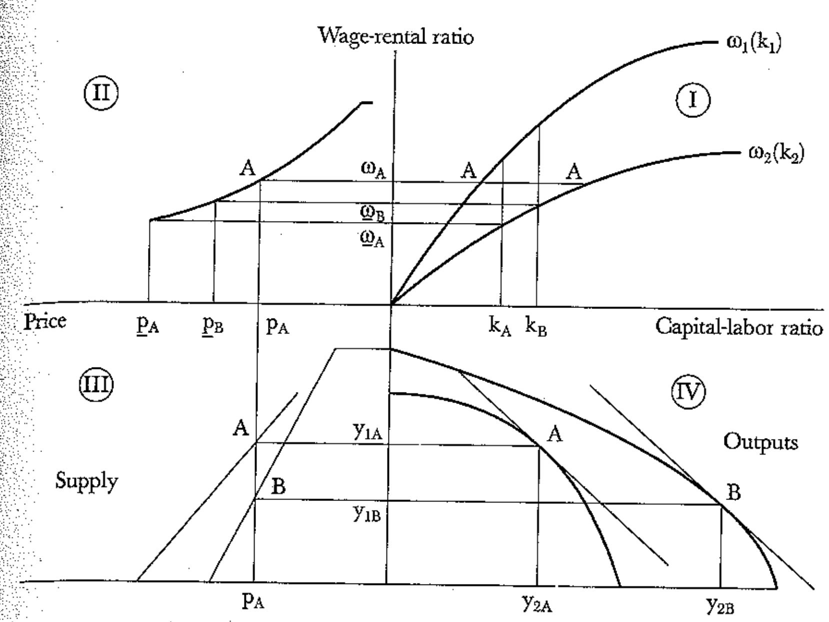

Figure 4.1 A supply shift due to a resource change

The last term is negative as ℓ is the labor share for the labor-intensive sector. A similar expression is obtained for sector 2 (the capital-intensive sector) with $(1 - \ell)$ replacing ℓ.

The changes just considered can be presented in terms of the shift of the transformation curve and the supply functions. In panel IV of Figure 4.1, the initial output corresponding to k_A and p_A is at A, and the output under $k_B > k_A$ and p_A is at B. The important thing to note is that B is in the southeast quadrant with respect to A. Panel III shows the corresponding shift in the supply from y_{1A} to y_{1B}. Thus, holding prices constant, the supply clearly depends on factor endowment.

Comparative Advantage: Heckshe-Ohlin

Basically, the foregoing discussion is a statement of the essence of the Heckshe-Ohlin theorem, which relates the trade pattern to the relative factor endowment. A "large" capital-labor ratio implies a "relatively large" production of the capital-intensive good and a "relatively small" production of the labor-intensive good. There are no absolute large or small quantities. The statement is comparative in nature, and it should be. The consequences for trade are drawn in reference to a simple world consisting of two countries that differ in relative resource endowment, employing the same technology. Trade results in a single world price faced by both countries. In this case the capital-abundant country exports the capital-intensive product and imports the labor-intensive product. Thus the capital-abundant country is said to have a comparative advantage in the production of the capital-intensive good. The orders of magnitude of the traded quantities depend of course on demand, but the discussion of demand can be postponed without affecting the nature of the argument. Originally, the concept of comparative advantage was developed by Ricardo to indicate differences in the costs of production due to differences in technology. We return to this below, but for now we note that the link between the two is related to the dependence of prices on resource endowment, which is taken up in the next section.

The dependence of supply on factor endowment can be applied to the evaluation of the growth pattern of a given country. As capital is accumulated, the country will increase the production of the output of the capital-intensive good. If the scope is broadened to include land, it is anticipated that land-abundant countries will tend to concentrate in the production of land-intensive products, such as grains. Indeed, countries with high land-labor ratios are big grain producers. Yet grain is also produced in countries with high labor-land ratios, such as India and China. India has turned from importing to self-sufficiency, even accumulating some surpluses. This achievement indicates that the foregoing considerations are not sufficient to guide us to the data. One of the reasons is that in this analysis it is assumed that the various countries use the same technology, but this need not be the case. The assumption of constant technology serves the purpose of isolating the effect of resource endowment on output composition and trade. However, it is always important to resist the temptation of identifying an assumption with evidence. Indeed, countries use different techniques in their production. In fact, we shall see later that the technology itself may depend on the resource endowment and that such a dependency affects the outcome.

Admissible Prices

Changes in resource endowment cause changes in the set of feasible prices. The nature of the shift is examined by observing the changes in the boundaries of the admissible prices. As a result of the monotonicity of the $\omega_i(k_i)$ functions and the full employment conditions, the boundaries of the admissible ω are monotonically increasing with k. If we assume that $\omega_i(k_i) \to \infty$ as $k_i \to \infty$, then for all $k > \bar{k}(\omega)$, ω is nonadmissible.[1] This means that under constant technology, capital deepening eventually forces any given wage-rental ratio to be too low to be admissible.

Capital deepening increases the wage relative to rental on capital, and consequently, the cost of production of the labor-intensive product increases relative to that of the capital-intensive product. Thus, in the process, any arbitrary price eventually becomes nonadmissible, and we have $\underline{p}(k) = p(\underline{\omega}(k))$, $\underline{p}'(k) > 0$.

PROPERTY 4.2 (admissible prices) Capital deepening raises the lower bound of the admissible wage-rental ratio and with it the lower bound of the admissible price of the labor-intensive product.

The dependence of prices on k is shown in panels I and II of Figure 4.1 for the change in the lower bound: $\underline{p}_B \equiv p[\underline{\omega}(k_B)] > \underline{p}_A \equiv p[\underline{\omega}(k_A)]$. Assume that initially the price is p_A, then there is a value of k, say $\bar{k}(p_A)$, such that for all $k > \bar{k}(p_A)$, p_A is inadmissible because $\underline{p}[k > \bar{k}(p_A)] > p_A$.

The price function $p(\omega)$ is invariant to changes in k that affect only its boundaries. To see this, recall that $d \ln p / d \ln \omega = S_{1L} - S_{2L}$, but $S_{iL}/1 - S_{iL} = \omega/k_i(\omega)$; hence S_{iL} is determined uniquely and independently of k. We can now relate the comparative advantage à la Hecksher-Ohlin with the cost of production by asking why the country moves to the Rybczynski point as k changes. Suppose that there is an increase in the capital-labor ratio, but the country tries, by inertia, to maintain a production pattern similar to the one prevailing prior to the change. The exact location of this point of inertia is purposely kept vague so that it can be applied to any point on the new transformation curve that lies in the northeast quadrant with respect to the initial point. All the points in the northeast quadrant are to the left of the Rybczynski point and therefore represent higher prices for the labor-intensive product compared with the initial ongoing world price. Thus even though this concept of comparative advantage is expressed in terms of resource endowment, it is rooted in the differences of costs of production. In the Hecksher-Ohlin framework, the cost of production associated with a given resource allocation changes because resources change.

As shown above, a change in k causes a change in output composition, a process that continues until $k = \bar{k}(p^*)$, at which point and beyond the country will specialize in the production of the capital-intensive good. This is the description of the product cycle that takes place in traded commodities when the price is determined in the world market. When a country is a price taker, as it accumulates capital, its production will shift from labor-intensive to capital-intensive products.

Summary

The summary of the supply conditions in Chapter 2 is now extended to cover the effect of resource change. For the labor-intensive sector we have

$$\underset{+\ -}{\ell(\omega, k)},\ \underset{+\ -}{\rho(\omega, k)},\ \underset{+\ -}{y_1(p, k)}.$$

The signs are permuted for the capital-intensive sector.

Accumulation and the Equilibrium Position

Small Open Economy

A change in resources produces a move from the initial equilibrium point to a new equilibrium point that we now try to characterize. From Property 4.1 it is clear that, under constant prices, an increase in k generates a decline in the production of the labor-intensive product. At the same time income goes up with k, resulting in an increased demand for the two products and generating an excess demand for the labor-intensive product. An open economy closes this gap through trade.

Figure 4.2 illustrates the changes for a price-taker country with an initial equilibrium at A where there is no trade. As k increases from k_A to k_B, the output, under p_A, changes to B. This causes a downward shift of the supply of the labor-intensive product, $y_1(p, k)$, as shown in panel III of Figure 4.1 by the movement of the schedule with A to that with B. The change in the equilibrium condition is shown in Figure 4.2. Initially A is on the demand curve $x_1^d(p_A, x_2)$, and there is no trade. With the change in k, the demand is at C, and the country becomes an importer of agriculture, with an import of $x_{1C} - y_{1B}$ and an export of nonagriculture of $y_{2B} - x_{2C}$. Other cases can be shown in a similar way. Try, for instance, to show the case where a country changes from exporting to importing nonagriculture. The change can also be shown in terms of the restricted demand function of a small open economy, and this is assigned as an exercise.

We now derive the result in terms of the excess demand:

$$z_i(p, k) = x_i[p, y(p, k)] - y_i(p, k)$$

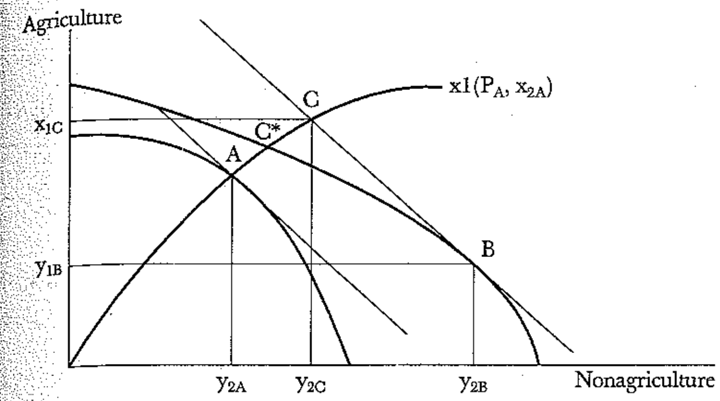

Figure 4.2 The impact of resource change: Small open economy

where $y(p, k)$ is income, evaluated at price p and resource level k. The sign of $\partial y_i/\partial k$ is determined by the factor intensity, but the response of total output, $\partial y/\partial k$, is always positive. The essence of this discussion is that the effect of a change in k on z_i is unambiguously signed for the labor-intensive sector, since for this sector

$$\frac{\partial z_i}{\partial k} = \frac{\partial x_i}{\partial y}\frac{\partial y}{\partial k} - \frac{\partial y_i}{\partial k} > 0.$$

In an open economy the excess demand is fulfilled by trade. Under balanced trade, the change in the value of import in one sector is equal to the change in the value of export of the other sector. Therefore the export of the capital-intensive sector increases with capital deepening. Hence

PROPERTY 4.3 (trade; expansion-pattern) The net import of the labor-(capital-)intensive product increases (decreases) with capital deepening.

A distinction should be made between a change in import and the corresponding change in the volume of trade. When the country is importing the labor-intensive product, an increase in net import implies an increase in the level of trade. On the other hand, if the country is exporting the labor-intensive product, capital deepening leads to a decline of trade.

The empirical validity of this discussion is straightforward but not simple to uncover without a detailed analysis, because time-series data reflect changes in resources, prices, and technology. This may be the reason for insupportable assertions that are sometimes made on the importance of increasing food import in generating overall economic growth. It is therefore useful to emphasize the partial result established here. Other things being equal, capital deepening will increase net import and decrease production of agriculture when it is labor intensive. Conversely, agricultural output will increase and net import will decline when agriculture is capital intensive. The implication of this for actual trade depends on whether the country is importing or exporting the labor-intensive product

If we assume that all countries accumulate capital, it is inconceivable that all of them will increase the net export of the capital-intensive good. To set the world trade in equilibrium, the price of the capital-intensive good will have to decline The situation is similar to that of a closed economy, to which we now turn

In passing it is noted that under equal factor intensities, the Rybczynski proposition does not apply. If the country produces the two products, there is no trade. The change in demand associated with capital deepening will be determined solely by the income elasticities

Closed Economy

The discussion of the closed economy is conducted with reference to Figure 4.3 The excess demand for agriculture at p_A generated by increasing k to k_B is given by the difference between the demand at C^* and the production at B. The gap is closed by increasing p, so that a new equilibrium point is found at E We have thus established

PROPERTY 4.4 (price path; closed economy) Capital deepening raises the wage-rental ratio and the relative price of the labor-intensive product.

While the change in prices caused by accumulation is uniquely determined, this is not the case with respect to outputs and therefore resource allocation Recall that $\ell(\omega, k)$ increases with ω and declines with k. Thus the resource allocation depends on the strength of the change in ω with respect to k, which in turn depends on the change in p, and this in turn depends on demand.

The increase in p called for by Property 4.4 generates a substitution effect in demand in favor of the capital-intensive product. Hence the income and substitution effects have the same sign for the capital-intensive product and contradicting signs for the labor-intensive product Therefore the equilibrium consumption of the capital-intensive product increases unambiguously, whereas the change in the consumption of the labor-intensive product depends on the relative strength of the substitution and income effects. Thus the only restriction on point E in Figure 4.3 is that it is to the east of A, meaning that

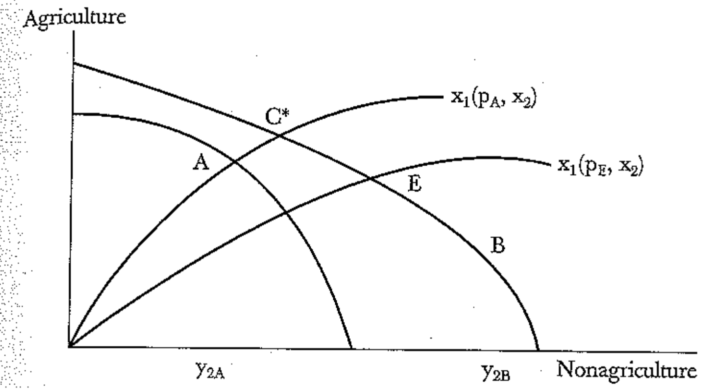

Figure 4.3 The impact of resource change: Closed economy

the labor-intensive product may increase or decrease The discussion is now summarized:

PROPERTY 4.5 (output path; closed economy) Capital deepening raises the per capita output of the capital-intensive sector, whereas the direction of the change in the output of the labor-intensive sector is ambiguous. It depends on the relative strength of the income and substitution effects on the demand side and the change in product composition on the supply side.

The importance of the demand elasticities is illustrated in terms of Figure 4.3. The income elasticity is reflected in the position of C^* with respect to A; the lower the income elasticity is, the closer x_{1C^*} will be to x_{1A}. The price elasticity is reflected in the degree of the rotation of the demand curve caused by a given change in p. A low price elasticity will cause E to be close to C^*, and thus the income effect will dominate. Alternatively, a high price elasticity will bring E closer to B and possibly lead to a decline in agricultural consumption.

Empirical Implications

The importance of the foregoing result, particularly the dependence of sectoral growth on income elasticities, cannot be overemphasized. It is all related to Engel's law, which states that the proportion of the consumer's budget spent on food declines with income. In other words, the utility function is not homothetic, and the income elasticity for food is less than one. The effect of the income elasticity is observed in Figure 1.4, which shows that in most countries the rate of growth of agricultural output was lower than that of total output, in spite of the decline in the relative price of agriculture. With all the necessary refinements and caveats, it is clear that faster growth in agriculture is concomitant with faster growth in nonagriculture. This is simply the reflection of consumers' preferences.

Sectoral Inputs

We turn next to an examination of the changes in factor employment along the equilibrium path. For a small open economy the price, and hence the factor-price ratio, are given. The response of the economy to an increase in k occurs completely through changes in output composition. The capital-intensive sector increases its output, and its employment of both capital and labor will increase. The labor-intensive sector will lose both labor and capital. In the closed economy part of the adjustment is made through the price. The increase in the wage-rental ratio encourages the substitution of capital for labor. By assumption, however, all the available labor has to be employed. Therefore this substitution is restricted by the full employment condition as well as by the need to satisfy the demand for the final products. In what follows we examine employment under these conditions, trying to separate the induced changes to various effects.

The possibilities in the adjustments of sectoral inputs are illustrated in Figure 4.4. To simplify the presentation here, we omit the sectoral notation. Point A represents the initial equilibrium position, where Y_A is produced with

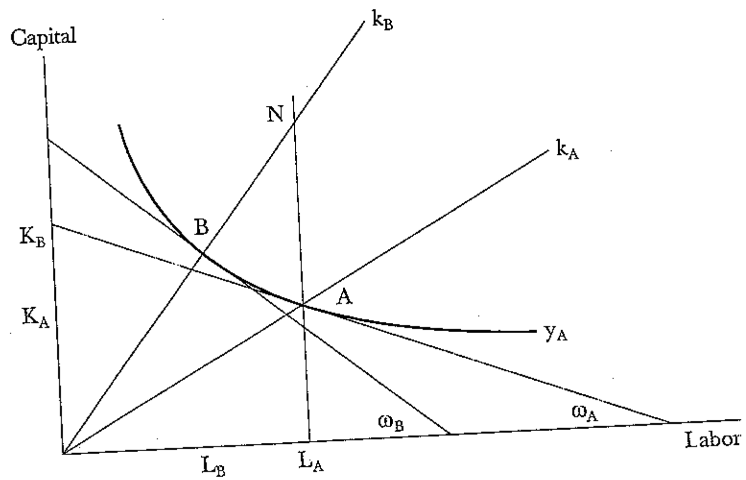

Figure 4.4 Intrasector resource allocation

L_A units of labor and K_A units of capital, yielding a capital-labor ratio of k_A and a wage-rental ratio ω_A. Point B is the least-cost input combination that produces the initial output Y_A under ω_B. We can refer to the decline in sectoral employment, $L_A - L_B$, as the factor substitution effect of employment. Similarly, $K_A - K_B$ represents the factor substitution effect of capital use.

The change in the actual sectoral employment depends on the strength of the expansion effect relative to the substitution effect. In terms of Figure 4.4, let N represent the point where the two effects are equal so that the initial labor input, L_A, is optimal under the new prices. The new equilibrium is on a ray through N. If it is below N, such as B, the sectoral labor input declines, and conversely, when it is above N, the sectoral labor input goes up. The situation is different with respect to capital, where the factor substitution effect supplements the expansion effect. We can then summarize the discussion by presenting the various effects along the equilibrium path generated by an increase in the capital-labor ratio:

Effect	Labor-intensive sector		Capital-intensive sector	
	Labor	Capital	Labor	Capital
Product expansion	+	+	+	+
Product substitution	−	−	+	+
Factor substitution	−	+	−	+

Of the four cases, only one is uniquely signed. The employment of capital in the capital-intensive sector increases with capital deepening. In all other cases, the final outcome depends on the relative strength of the various effects. It should, however, be noted that the capital-labor ratios increase in both sectors, that is, $(\hat{k}_i/\hat{k})\,|_{eq} > 0$, where eq stands for changes along the equilibrium path. This simply follows from the fact that $(\hat{\omega}/\hat{k})\,|_{eq} > 0$.

More can be said when we know the relative strength of the various effects. For instance, if the income elasticity of agriculture is negligible, then the product substitution effect exceeds the product expansion effect. In the limiting case of zero income elasticity for agriculture, there is no product expansion in agriculture. When agriculture is labor intensive, its labor will decline. If the product substitution effect is also negligible, then K_1 will increase.

Alternatively, if the product substitution effect is negligible and that of income is not, then the sectoral capital will definitely increase, whereas the change in labor depends on the relative strength of the income elasticity and the factor elasticity of substitution. Finally, when the factor elasticities of substitution are small, the changes in the final demand will determine the changes in factor employments. It is therefore evident that the final outcome depends on the relative magnitude of the demand elasticities and the elasticities of substitution and on factor intensities. The impact of capital deepening on factor use in agriculture can thus be summarized:

PROPERTY 4.6 (input path; closed economy) Capital deepening raises the capital-labor ratio in both sectors and the use of capital in the capital-intensive sector. When the demand elasticities for agriculture are small, the main changes in inputs are supply driven. Thus when agriculture is labor intensive, it is likely that its total use of capital will increase, whereas labor employment will decline.

Price-Responsive Factor Supply

The foregoing analysis assumed factor supply to be perfectly inelastic. This assumption is rather restrictive and can be relaxed by allowing factor supply to depend on their prices (Frenkel and Razin, 1975). The consequences of this generalization are examined below. Chapters 9 and 10 consider the determinants of factor supply.

Labor

We now introduce the assumption that labor supply responds positively to the wage rate, $L'(w) > 0$. In evaluating labor supply, it is important to distinguish between long-term effects and cyclical responses. The empirical literature on labor supply does not give a clear indication of the numerical value of the labor-supply elasticity. Some of the development literature assumes, at least implicitly, that labor supply is price responsive to the food wage, w/p. In the present framework, w (measured in terms of nonagriculture) and w/p (measured in terms of agriculture) move in the same direction, and therefore there is no particular advantage in dealing specifically with the food wage.

In what follows we differentiate between labor supply and population, N, which is taken to be exogenous. The supply side of the model is now modified by restating the full employment conditions:

$$L(w) = L_1 + L_2, \quad L'(w) > 0,$$
$$K = K_1 + K_2,$$
$$L(w) \leq N.$$

The labor supply also depends on the population and on other demographic variables, such as the age distribution, that are taken here to be exogenous. These variables are assumed to be constant in the analysis and therefore are not included explicitly as arguments of $L(w)$.

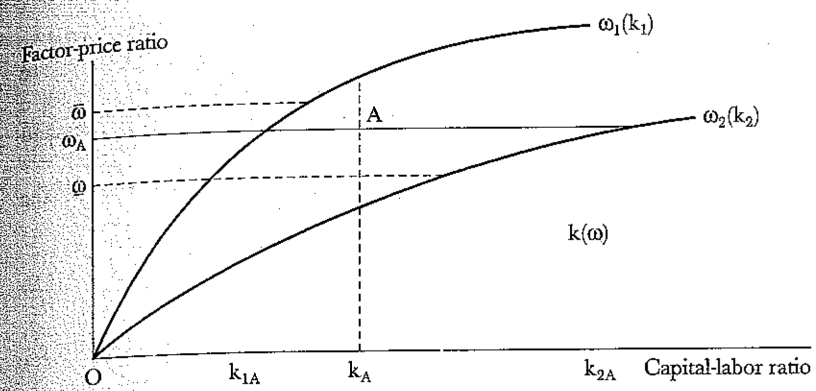

Figure 4.5 Admissible factor-price ratios

The production functions (2.3) and the competitive conditions (2.4 and 2.5) remain unchanged. The overall capital-labor ratio is now a declining function of wage,

$$k(w, K) = K/L(w), \quad \frac{\partial k(w, K)}{\partial w} < 0.$$

This property can be expressed in terms of the wage-rental ratio. Recall that w/p_2 is monotonically increasing in k_2. When p_2 is the numeraire, $w/p_2 = w$, and therefore $w(k_2)$ is monotonically increasing in k_2. Combining this with the monotonicity of $k_2(\omega)$, we can write $k(\omega, K)$, $\partial k(\omega, K)/\partial\omega < 0$. Let $\ell = L_1/L(\omega)$, and write the full employment condition:

$$\ell(\omega, K) = \frac{k_2(\omega) - k(\omega, K)}{k_2(\omega) - k_1(\omega)}$$

The determination of the resource allocation is shown in Figure 4.5 where k is now a declining function of ω. The admissible values of ω are bounded by $\underline{\omega} = \min(\,)$ and $\overline{\omega} = \max(\,)$, where $(\,) = [\omega_i(k(\omega_i, K))$, $i = 1, 2]$. For any admissible ω, k is determined by $k = k(\omega, K)$. Thus k_A is the capital-labor ratio corresponding to ω_A, and similarly $k_{iA} = k_i(\omega_A)$. Per capita outputs are obtained from

$$y_i = \frac{L(\omega)}{N}[\ell_i(\omega, K) f_i(k_i(\omega))]. \tag{4.1}$$

The correction factor for the participation rate in the labor force, $L(\omega)/N$, increases in ω. Given ω, factor prices and the product price ratio, p, are determined as before. This concludes the solution of the supply side for a given ω, where the equilibrium value is determined by introducing the demand function.

To analyze the behavior of the system in terms of ω, we first examine the labor allocation:

$$\frac{\ell(\omega, K)}{1-\ell(\omega, K)} = \frac{k_2(\omega) - k(\omega, K)}{k(\omega, K) - (k_1(\omega)} \tag{4.2}$$

Since $k_i'(\omega) > 0$, $\partial k(\omega, K)/\partial\omega < 0$, and $k_1(\omega) \leq k(\omega, K) \leq k_2(\omega)$, the expression in (4.2) is monotonically increasing in ω. Consequently,

$$\partial\ell(\omega, K)/\partial\omega > 0. \tag{4.3}$$

As ω increases, k declines, whereas k_1 and k_2 increase and therefore ℓ increases. Consequently, $\partial[\ell(\omega, K) f_1(k_1(\omega))]/\partial\omega > 0$, and therefore,

$$\partial y_1(\omega, K)/\partial\omega > 0 \tag{4.4}$$

Using the definition of ℓ, it follows that L_1, the employment in the labor-intensive sector, increases with ω. Since also $k_1'(\omega) > 0$, it follows that $K_1(\omega)$ increases in ω, and because K is fixed, $K_2(\omega)$ must decrease with ω. But since $k_2'(\omega) > 0$, it follows that $L_2(\omega, K)$ declines in ω Consequently the capital-intensive sector employs fewer resources, and its output declines. It is possible to view this change in two steps. First, an exogenous increase of wage increases labor supply. The second step is basically the Rybczynski effect, which favors the labor-intensive sector.

The analysis of the supply side can be summarized by the transformation curve. This curve is determined by $k(\omega, K)$ and is denoted as $T(k(\omega, K))$. As such it differs from the transformation curve obtained under fixed resources, $T(k)$. The relationship between the two transformation curves is discussed below.

Introducing a demand curve allows the determination of the equilibrium point for the economy in the same way as before. Note, however, that in the present case, the level of employment in the economy is determined jointly with the other dependent variables.

Price-Responsive Capital Supply

A similar analysis follows for the case where the supply of capital depends on the rate of return. This relation may represent the positive effect that the rate

of return may have on saving. Alternatively, we can think of a country that borrows from abroad and is faced with an upward-sloping supply function of capital. Since the present analysis is timeless, we can express the capital stock (rather than investment) as dependent on r. Thus the full employment conditions will be

$$K(r) = K_1 + K_2, \quad K'(r) > 0,$$
$$L = L_1 + L_2.$$

The analysis is similar to the analysis with varying labor supply. Since r declines in k_2 and k_2 increases with ω, we have $r'(\omega) < 0$, and for fixed L, $k(\omega, L) = K[r(\omega)]/L$, $\partial k(\omega, L)/\partial\omega < 0$.

It is now possible to describe the present case in terms of Figure 4.5. To any feasible ω corresponds a rate of return $r(\omega)$ that determines the level of the capital stock, which in turn determines k for a fixed labor force. Resource allocation is determined conditional on ω, and this in turn determines outputs. Thus a parametric solution of the system conditional on ω generates the transformation curve. Again, as in the previous case, the introduction of demand makes it possible to determine the equilibrium point. In this case, the level of capital is determined jointly with the other dependent variables. The exogenous increase in the rental rate increases the capital stock and consequently causes an expansion of the capital-intensive sector and a shrinkage of the labor-intensive sector.

A Generalization

The analysis can now be generalized by combining the two cases discussed above where the overall capital-labor ratio is a function of ω alone: $k(\omega) = K(\omega)/L(\omega)$, $k'(\omega) < 0$. The foregoing analysis is immediately applicable to this more general case. Total employment and the level of the capital stock are now determined along with the other dependent variables

We begin the analysis by comparing the production possibilities under fixed resources, $T(k)$, with those under varying factor supplies, $T(k(\omega))$. Since the labor force is not constant anymore, the outputs are measured in per capita, rather than per worker, terms. We return to this point below. In Figure 4.6, $T(k_A)$ is drawn for the capital-labor ratio k_A, which under varying factor supply corresponds to $k_A = k(\omega_A)$. Point A on this curve corresponds to ω_A, whereas point B corresponds to $\omega_B < \omega_A$. At B, the output of the labor-intensive sector is lower than at A.

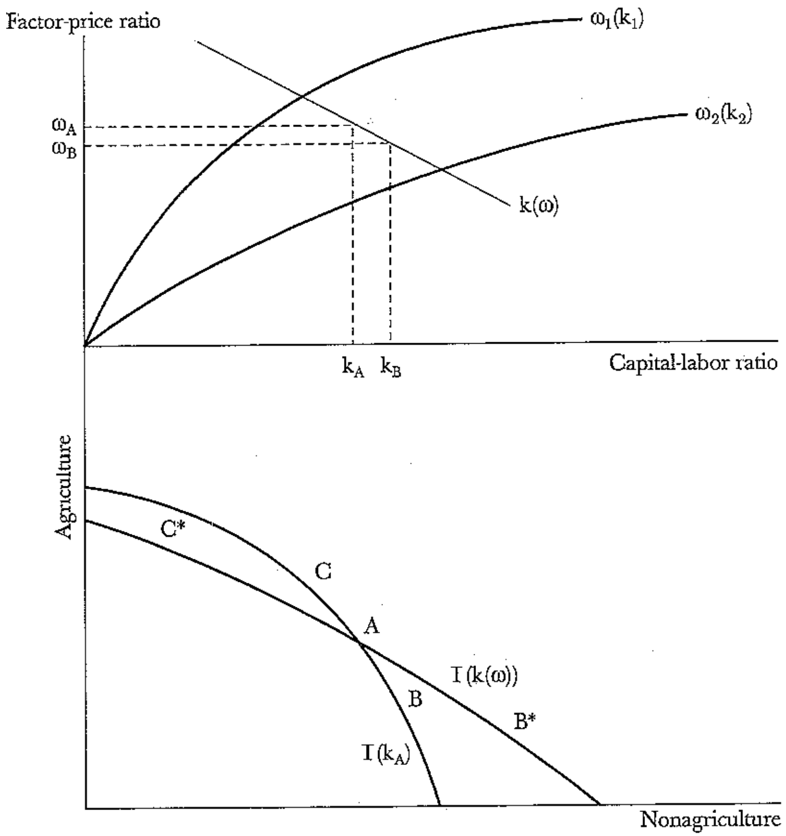

Figure 4 6 Price-responsive factor supply

Turning to $T[k(\omega)]$, we note that by construction point A is also located on this curve. However, a decline in ω increases k so that at $k_B = k(\omega_B) > k_A$ output is at B^* on $T(k_B)$. B^* is a Rybczynski point, located in the southeast quadrant with respect to B, reflecting an increase in the output of the capital-intensive sector and a decrease in the output of the labor-intensive sector The question is whether B^* is outside or inside $T(k_A)$ The answer depends on the direction of the resource change. When only L changes, then with $\omega_B < \omega_A$, $L(\omega_B) < L(\omega_A)$, and B^* will be inside $T(k_A)$. When only K changes, then $k(\omega_B) > k(\omega_A)$, and therefore B^* is outside $T(k_A)$, as shown in Figure 4.6.

A similar analysis can be conducted for $k_C = k(\omega_C) < k_A$, $\omega_C > \omega_A$. The resulting point C^* on $T[k(\omega)]$ is in the northwest with respect to C on $T(k_A)$. Again, the location of C^* with respect to $T(k_A)$ depends on whether the change is in L or K. If only labor changes, $L(\omega_C) > L(\omega_A)$, and therefore C^* will be outside $T(k_A)$. The opposite is true when only K changes in response to a change in ω In this case K declines and C^* is inside $T(k_A)$ as shown in Figure 4.6

When the two factors are price responsive, a change in ω causes opposite effects on the quantities supplied of the two factors For instance, an increase in ω causes a rise in the real wage and hence in the labor supply. At the same time the real rate of return declines and therefore K declines. The net effect on total output may be positive or negative depending on the supply elasticities and the productivity of the two inputs

This framework makes it possible to prove that the product supply functions are more elastic the more elastic are the factor supply functions (Frenkel and Razin, 1975) This topic is discussed in some detail in Chapter 14 Initially, $T[k(\omega)]$ and $T(k_A)$ intersect at A. Let $\omega_B = \omega_{B*} < \omega_A$, and $p(\omega)$ is the price of the labor-intensive product, y_1 in our case. Consequently, $p'(\omega) > 0$ and $p(\omega_B) = p(\omega_{B*}) < p_A$. Since B^* is in the southeast quadrant with respect to B, we have shown that reducing p causes a stronger response of Y_i when the factor supplies are price responsive.

PROPERTY 4.7 (product and factor supply elasticities) The product supply elasticities monotonically increase with the factor supply elasticities

One outcome of the present analysis not commonly appreciated is that the supply price is not equal to the slope of the tangent line to $T[k(\omega)]$.[2] This follows from the fact that any point on $T[k(\omega)]$, say A, is also located on some $T(k)$, in this case $T(k_A)$, which intersects $T[k(\omega)]$ at A. Since p_A can be derived as the slope of the tangent line to $T(k_A)$, the tangent line to $T[k(\omega)]$ must have a different slope. Under fixed factor supply we have $|dy_2/dy_1| = p$ This represents the concept of real marginal cost, a trade-off between two outputs. Under varying factor supplies, the movement along a transformation curve implies a change in the factor quantities, and there is no pure trade-off [3]

The equality of $|dy_2/dy_1|$ and p is often used to derive the concavity of the transformation curve. The equality, however, does not hold for $T[k(\omega)]$. In any case, the concavity, or lack of it, is not of immediate relevance to the discussion. What matters here is that the supply function is positively sloped and that $Y_1(Y_2) = Y_1[Y_2 : (Y_1, Y_2) \in T(k(\omega))]$ is ever declining in Y_2. With these properties, the equilibrium is uniquely determined To summarize the supply conditions, we note

PROPERTY 4.8 (supply determinants) The product supplies are functions of the product price ratio and the factor supply functions.

Given the factor supply functions, inputs and outputs can be written as functions of the price alone. Thus for the labor-intensive sector, we have $\partial \ell(\omega \mid \cdot)/\partial \omega > 0$, $\partial y_1(p \mid \cdot)/\partial p > 0$, where $\mid \cdot$ indicates holding factor supply *functions* (and of course technology) constant, and $p = p(\omega)$ is the price of the labor-intensive product. The signs for the capital-intensive sector are reversed. On the surface, this result is very strong in the sense that the product supply functions are not constrained by the available resources, because those can change subject to their supply functions. In an economy that behaves according to this model, including constant technology, the data should reveal a positive relationship between output and price, with no other variables added to the equation. However, this is only true as long as the factor supply functions remain constant. This restriction limits the usefulness of this result for empirical analysis using cross-country data because the factor supply functions are not the same in the various countries. Thus the model is merely useful as a reference and not as a final formulation.

Equilibrium Analysis

The outcome of the foregoing analysis is that the production structure that was used for fixed factor supply can be used also for varying supply, with a proper interpretation that takes into account the dependence of the transformation curve and the supply functions on the factor supply functions rather than on the factors themselves. The feasible prices are well defined as before. Hence, if the factor supply functions give finite quantities for finite prices, then the transformation curve represents finite outputs that constitute the efficiency frontier. The supply of each of the products varies positively with its own price. Consequently the equilibrium analysis can be conducted in the same way as in the fixed supply case and need not be repeated. The main point here is that an equilibrium output is well defined.

Perfectly Elastic Factor Supply

To complete the discussion of factor supply we now consider the limiting case of perfectly elastic factor supply, where the factor supply is neither price responsive nor exogenously given. This can be thought of as the case of a small open economy that can borrow at a constant rate. This assumption is often used in the discussion of the real exchange rate for a small open economy (Obstfeld and Rogoff, 1996, chap. 4).

Suppose that there is a perfectly elastic supply of capital at a rental rate r^0. The full employment condition now binds only the labor allocation, whereas capital can be used at any desirable level. Knowing r^0, the system is solved for k_i, ω, w, and p, all of which are now constants. Labor allocation, as given by $\ell = (k_2^0 - k)/(k_2^0 - k_1^0)$, is now a function of k alone, but k depends on K, which is undetermined without knowing the demand. Per capita sectoral output is $y_i = f_i(k_i^0)\ell_i(r^0, k)$. The transformation curve is generated by changing the value of ℓ from zero to one. It is a straight line with slope

$$\frac{dy_2}{dy_1} = \frac{f_2(k_2^0)}{f_1(k_1^0)} \frac{d(1-\ell)/dk}{d(\ell)/dk} = -\frac{f_2(k_2^0)}{f_1(k_1^0)}.$$

The slope of the transformation curve is not equal to the price. The cost of production associated with each worker in the ith sector is $w^0 + r^0 k_i^0 = \zeta_i^0$; $\zeta_i^0 = \zeta_i(r_i^0)$, and under zero profit it is equal to the average value output per worker, $p_i f_i(k_i^0)$. Hence

$$p^0 = \frac{p_1}{p_2} = \frac{\zeta_1^0/f_1}{\zeta_2^0/f_2} = -\frac{dy_2}{dy_1}\frac{\zeta_1^0}{\zeta_2^0}.$$

gain, the reason for the inequality of the functions p and dy_2/dy_1 is that K varies along the transformation curve, and as such the slope represents an expansion effect in addition to a trade-off.

When the price is constant, the supply is perfectly elastic for all feasible allocations, and the production is determined by demand. In the case of the small open economy, this implies specialization: if $p^* > p$, the economy will specialize in agriculture and conversely for $p^* < p$. In the case of equality, there is no unique determination of output composition. In the case of a closed economy, the equilibrium is determined by the intersection of the demand and the transformation curve. The equilibrium determines outputs, which in turn determines the resource allocation, which in turn determines k, which in turn determines K.

PROPERTY 4.9 (perfectly elastic factor supply) When the supply of one of the factors is perfectly elastic, the price is constant, and the product supplies are also perfectly elastic. In a closed economy the output composition is determined by demand.

How instructive is this model? Because we do not see wide fluctuations in the production of agriculture and nonagriculture, it appears that this model is not very realistic. There may be other reasons, however, for the lack of wide fluctuations in outputs. First, we note that this is a long-run model, and by the nature of things the response is slow, though not necessarily weak (see Chapter 14) Second, as countries borrow heavily, the risk of their default increases; therefore the cost of borrowing has to increase accordingly, which leads to the case of less than perfectly elastic capital supply. Third, to speak of a small open economy in this context overlooks the fact that some products are nontradable This brings us to the dichotomy of tradable and nontradable sectors In this case, the price is the real exchange rate. The output composition is determined by the domestic demand, and hence there is no tendency for specialization in production. Fluctuations in domestic demand will be absorbed by net import, and the real exchange rate is stabilized at a level determined by the cost of borrowing. An interesting message comes out of this discussion; elastic supply of capital helps to stabilize the real exchange rate in this artificial world

Obviously, this framework is oversimplified, and specifically, it overlooks the dynamic aspect of the problem The derived net import of capital is determined by the intersection of the demand curve with the transformation curve and by available domestic capital. If the gap is big, the country will be a big importer of capital. Such a situation may not be very stable, and therefore may not exist for a long time period Changes in the supply of capital will cause changes in the real exchange rate

We have concentrated on capital, but the stabilization comes also from a perfectly elastic supply of labor. Indeed, many affluent countries depend on migration to supplement the domestic labor supply. This international migration contributes to the stabilization of wages (or to slowing down their increase) In fact, labor migration may be more effective as a stabilizer than is capital inflow. When the economy turns sour, international capital often flies out at the speed of electronic transactions, whereas labor tends to stay in the country

For the sake of completeness, we mention also the case where the supply of both factors is perfectly elastic. Consequently, the supply of each of the products is perfectly elastic, and the demand determines the level of production in each of the sectors. The important thing here is that the production side of each sector can be analyzed independently of the other sector. This case is applicable when one of the sectors is relatively small, and it can be considered as a price taker in all the pertinent factor markets.

Extensions

The Equilibrium Path under Tax

The question is what changes, if any, should be made in the characterization of the equilibrium path if the economy is operating under a tax-subsidy on

products. To answer this we note that with p measuring the supply price, the supply function $y_1(p, k)$ is unaffected by the tax, and therefore the shift of $y_1(p, k)$ caused by k is independent of the tax. Introducing a restricted demand function under distortion, as in Figure 3.1, facilitates the determination of the equilibrium solution. Accordingly, capital accumulation decreases the supply of the labor-intensive good, and therefore its price increases. The case of an open economy is even simpler It can thus be concluded that the qualitative behavior of the system in response to changes in k is the same as that observed under no tax.

Also, allowing for varying factor supply does not change the qualitative results The response to price changes caused by the tax will be stronger, however, because the supply elasticity is larger than that obtained for fixed resources For instance, subsidizing agriculture increases the agricultural prices, and therefore a stronger supply response is generated. As a consequence the equilibrium price will be lower and output higher as compared with fixed resources.

There are some additional aspects to the behavior of the real exchange rate as a result of resource change, but they bear similarity to the effect of productivity changes; therefore we defer the discussion on these to Chapter 5

Aid and Resource Change

Capital deepening increases the net import of the labor-intensive product. If agriculture is labor intensive, and the country is an importer of food, its import will grow with capital deepening. Food aid to such a country can supply some of the augmented demand Thus it is possible to have simultaneous growth in food aid and food import. This is consistent with the conclusions drawn in Chapter 3 that aid replaces commercial imports. It is a mistake, however, to attribute the increase in commercial import to food aid

Although emphasis is placed on capital accumulation, we should pause to consider the case where labor grows faster than capital, generating population pressure on given resources. This is still the case in some low-income countries that, largely due to political conflict, are unable to divert sufficient resources to investment. In such countries, a decline in k leads to an increase in the *per capita* supply of agriculture and a decline in the *per capita* import. This is a direct outcome of the analysis However, note that in this case *total* import, as well as total consumption, may rise. This will happen when the relative decline in per capita import is smaller than the rate of population growth Thus it may happen that the country jointly increases its food consumption, production, and import. In this case, some food aid will reduce the commercial import but not to the extent of showing a decline in import

In a closed economy, we find that the depressive effect of food aid on agriculture is stronger under varying factor supply, simply reflecting larger supply elasticities.

Price-Responsive Land Supply

As shown in Figure 1 5, the cultivated area varies over time with different rates across countries We now extend the discussion in Chapter 2 by allowing the size of the agricultural land to be determined endogenously. To do so, we introduce two new elements into the discussion, land quality and land-specific cost.

Land is a factor specific to agriculture, and therefore the rent on land is the meaningful measure of the terms of trade of agriculture (Mundlak, 1969). When the expected rent is positive, investment will be made in land expansion, and when it is negative, land will go out of production The expansion often requires investment in reclamation, bush clearing, roads, communications, or other infrastructure facilities. Such investments constitute set-up costs, and once made, they do not affect decisions on the intensity of land utilization On the other hand, there are costs incurred as a result of holding or cultivating the land, and as such they affect the decisions of whether and to what extent the land is utilized. It is therefore important to differentiate between the expansion and contraction of cultivated area.

We turn to examine the effect of the various determinants of land size and the relationships between the size and the intensity at which land is cultivated. This brings us to a consideration of the Ricardian extensive and intensive margins. The main determinants to be examined are factors affecting the profitability of agriculture, the resource constraints, the role of infrastructure, and tax. Technical change is taken up in Chapter 5. The discussion abstracts from all the institutional factors affecting farmers' behavior with respect to land, particularly those emerging from differences between ownership and operation.

The model is developed in steps in order to evaluate the role played by the main pertinent variables In each step the simplest version needed to make the point is presented. As indicated above, the cost of land may take different forms, affecting long-run and short-run decisions differently. We do not differentiate between them in the discussion, but the results can be interpreted according to the case of interest. To economize the discussion, cost will sometimes be referred to as tax, but this is a generic term to be interpreted according to the case at hand. The model assumes two factors of production, land and

Table 4.1 Summary results

Exogenous variables	Endogenous variables							
	$k(q)$	z	A	K	Y	R	r	p
			Agriculture is a price taker					
r/p	−	+	−	−	−	−		··
c/p	0	+	−	−	−	−	··	··
			Capital constraint					
K	+	−	+		+	+	−	··
c/p	+	+	−	··	−	−	−	··
			Output quota					
Y	+	−	+	+	··	+	··	+
r	−	−	+	−	··	+	··	+
c	+	+	−	+	··	−	··	+

Note: ·· Irrelevant

capital. The first version assumes agriculture to be a price taker, the second model assumes a capital constraint, and the third model assumes an output constraint. These three versions provide the needed insight and background for the empirical analysis. The effect of technical change is taken up in Chapter 5.

The results of the analysis are summarized in Table 4.1, which can be studied without working through the details of the developments.

Initial Specification

The starting point is the model used by Lucas (1978) to analyze the distribution of firms The quality of land, denoted by q, is assumed to be a continuous variable that takes on nonnegative values; the higher the value of q, the better the land Let $A(q)$ be the available amount of land of quality q. To normalize the units of quality, we can refer to the distribution of land by quality, as illustrated in the upper panel of Figure 4 7[4] The land used by agriculture, referred to as cultivated land, is all the land of quality $q \geq z$, where z is the marginal quality to be determined by the model. The area under cultivation is

$$A = \int_{z}^{\infty} A(q)dq. \tag{4.5}$$

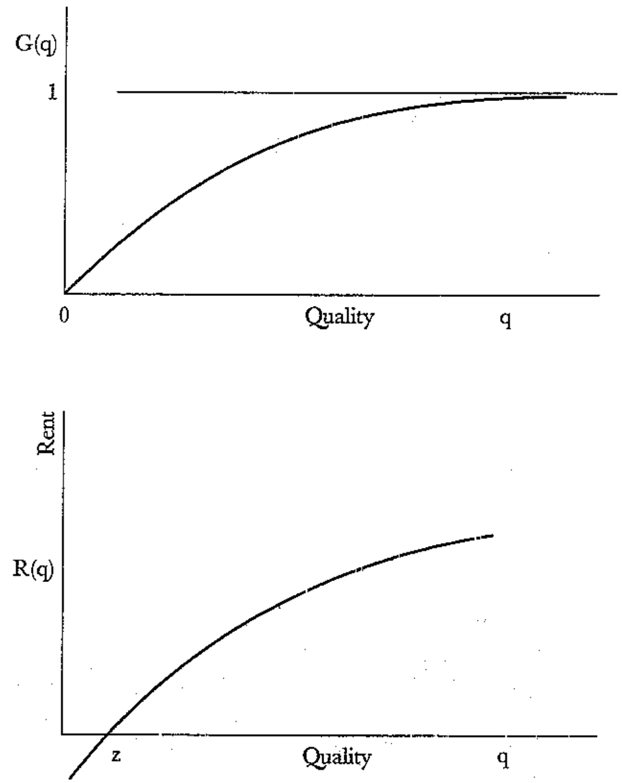

Figure 4.7 Distribution of land quality

The cultivated land, adjusted for quality is

$$Q = \int_z^\infty qA(q)dq \qquad (4.6)$$

To simplify the discussion, assume that there is only one factor of production in addition to land, labeled as capital.[5] $K(q)$ is the amount of capital allocated to quality q land, and $k(q)$ is the capital-land ratio, $k(q) = K(q)/A(q)$. Total capital used by agriculture is

$$K = \int_z^\infty k(q)A(q)dq \qquad (4.7)$$

To allow the marginal rate of factor substitution to depend on land quality, the production function is expressed in terms of land measured in quality units, $qA(q)$. Assuming CRS, the function for quality q land is

$$Y(q) = F[qA(q),\ K(q)] = A(q)qf[k(q)/q], \qquad (4.8)$$

where $qf(k/q)$ is the output per unit of quality q land. By assumption, the function $f(\cdot)$ is twice differentiable, increasing, and strictly concave, $f(0) = 0$, $f'(0) = \infty$, $f(\infty) = \infty$, $f'(\infty) = 0$. Total output is

$$Y = \int_z^\infty Y(q)dq = \int_z^\infty qf(k(q)/q)A(q)dq. \qquad (4.9)$$

The quantities A, K, and Y are functions of z and $k(q)$; these in turn are determined by the state variables, which vary with the specification of the model. Let the user price of capital (rental rate) be r, the product price be p, and c be the unit cost of using land, independent of its quality. The rent on quality q land is

$$R(q) = pqf[k(q)/q] - c - rk(q). \qquad (4.10)$$

Differentiating $R(q)$ with respect to q, we obtain $R'(q) = [R(q) + c]/q > 0$, implying that the rent increases monotonically with q. The marginal quality z is determined by $R(z) = 0$. Hence for all $q > z$, $R(q) > 0$, whereas the rent of the low-quality land, defined by $q < z$, is negative, and therefore that land is not cultivated. This is illustrated in the lower panel of Figure 4.7. The cost, c, plays an important role here because it forces low-quality land out of production.[6] The economic problem is to determine what land to cultivate and at what level of intensity, measured by $k(q)$. The different models considered below will endogenize some of the prices in equation (4.10).

Agriculture as a Price Taker

In this model it is assumed that there is a perfectly elastic supply of capital at the rental rate r and a perfectly elastic product demand at price p, and the cost c is independent of the land quality. In this specification, the level of intensity is determined independently of the extensive margin, z, because there is no resource or output constraint. Nevertheless, we will employ a general formulation that will serve us in the other cases. The formal presentation of the present problem is

$$\max_{k(q),z} L(k(q), z) = \int_z^{\infty} [pqf(k(q)/q) - rk(q) - c]A(q)dq \tag{4.11}$$

The first-order conditions for an internal solution (FOC) are

$$L_1() = pf'[k(q)/q] - r = 0, \tag{4.12}$$

$$L_2() = pzf[k(z)/z] - c - rk(z) = 0, \tag{4.13}$$

where L_1 and L_2 are the partial derivatives with respect to $k(q)$ and z respectively. In this model, the solution is recursive; we first solve from (4.12) for $k(q)$. We insert the solution to equation (4.13), compute the rent for all q, and determine z according to the second condition. The order of the solution cannot be reversed because the value of z does not affect $k(q)$. If we keep this in mind, the solution for z can be illustrated by drawing the graphs of the FOC in the (k, q) plane. Starting with equation (4.12), $f'(k(q)/q)$ is a monotone function; its inverse exists and we can rewrite it as

$$k(q) = q\Phi(r/p), \tag{4.14}$$

where $\Phi(r/p) = f'^{-1}(r/p)$, $\Phi'(r/p) < 0$. This function indicates the optimal intensity for all q, and is shown in Figure 4.8. Given r/p, $k(q)$ is a monotone increasing function of q, so that $k(q_A) < k(q_B)$ for $q_A < q_B$.

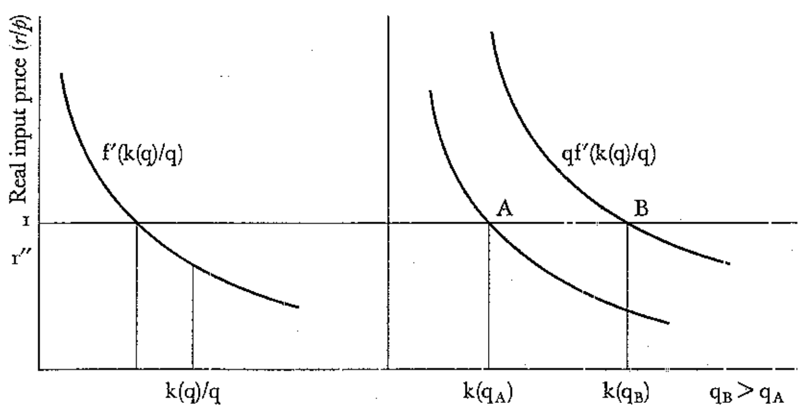

Figure 4.8 Input demand

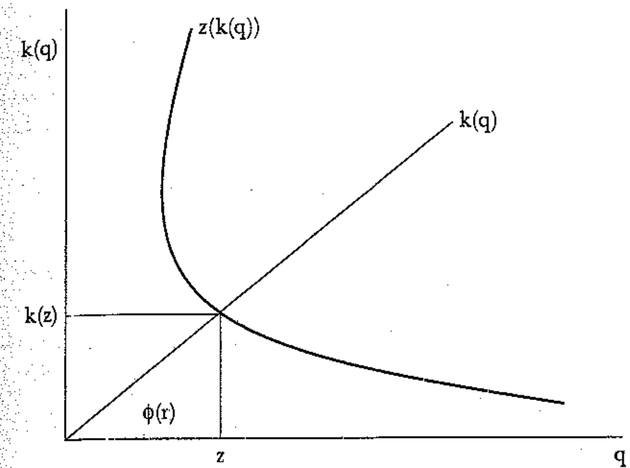

Figure 4.9 Intensive and extensive margins

To determine z, we obtain the graph $R(k, q) = 0$ which traces the combinations of k and q that result in zero rent. Using equation (4.13), we calculate the zero rent quality for any arbitrary k,

$$z(k) = \frac{rk}{pf(k/z)} + \frac{c}{pf(k/z)} \tag{4.15}$$

This graph is drawn in Figure 4.9. As k increases the second term on the right-hand side of equation (4.15) declines, and the first term increases due to the concavity of $f(k)$, so that the cost of capital increases linearly at rate r, but its marginal productivity decreases; in the limit, $k \to \infty$ implies $z \to \infty$. On the other hand, as k declines, the first term on the right-hand side declines due to the concavity of $f(k)$, whereas the second term increases at a faster rate; in the limit, $k \to 0$ implies $z \to \infty$. This means that to stay in production the quality must increase in order to compensate for the decline in output resulting from the decline in intensity of cultivation. Because resources are costly, producers are limited in their choice of k, and the optimal choice is given by equation (4.14). Thus the intersection of the two functions, (4.14) and (4.15), provides the solution for z and $k(z)$.

The Margins

How do the margins respond to changes in the state variables r, p, and c? To answer this question, we differentiate the first-order conditions. Without a loss in generality we set $p = 1$ so that r and c are measured in product units. When it is desirable to emphasize the role of p, we will switch notation and rewrite the real factor prices as r/p and c/p. To simplify the notations when ambiguity does not arise, we use in what follows $f(q) \equiv f(k(q)/q)$ and $f'(q) \equiv df(k(q)/q)/dk(q)$.

We differentiate equation (4.12), using equation (4.14), $f''(q)dk(q) = qdr$ (note that $k(q)$ is independent of c), to obtain for $f''(q) \not\equiv 0$

$$\frac{1}{q}\frac{dk(q)}{dr} = \frac{1}{f''(q)} = \Phi'(r/p) < 0. \tag{4.16}$$

Thus equation (4.16) is simply the condition that the demand for an input declines with its real price. For any pair (y, x), we label the elasticity of y with respect to x as $E(y, x)$ and rewrite equation (4.16) as elasticity:[7]

$$E(k(q), r) = \frac{d \ln k(q)}{d \ln r} \equiv -\sigma < 0,$$

where $\sigma > 0$ is a measure of the degree of concavity of the production function. The stronger the diminishing returns are, the smaller σ is, and therefore the weaker the relative response of capital intensity to a given change in r will be.

Turning to the marginal quality, differentiate (4.13) and rearrange, using the FOC:[8]

$$c/z\, dz = k(z)\, dr + dc \tag{4.17}$$

It then follows that $\partial z/\partial c = z/c > 0$ and $\partial z/\partial r = zk(z)/c > 0$. Writing the response as elasticities:

$$E(z, c) = 1, \tag{4.18}$$

$$E(z, r) = \frac{rk(z)}{c} = \frac{S(k; z)}{S(c; z)}, \tag{4.19}$$

where $S(k; q) = rk(q)/qf(q)$ and $S(c; q) = c/qf(q)$ are the factor shares of capital and tax in the output of quality q land. At the margin, where $q = z$, the two shares add up to 1.

Equations (4.18) and (4.19) indicate that the marginal quality increases with r and c and therefore declines with p. This is illustrated in Figure 4.10. A decline in c moves the graph of equation (4.15) downward from $R = 0$ to $\tilde{R} = 0$, and the solution moves from B to C with lower z and $k(z)$. However, as $\Phi(r)$ is unchanged, $k(q)$ remains constant for every q. On the other hand, a decline in r also moves the graph downward, but it also changes $\Phi(r)$ so that $k(q)$ increases for every q. Consequently, the solution moves from B to N with lower z and higher $k(q)$ for all admissible q

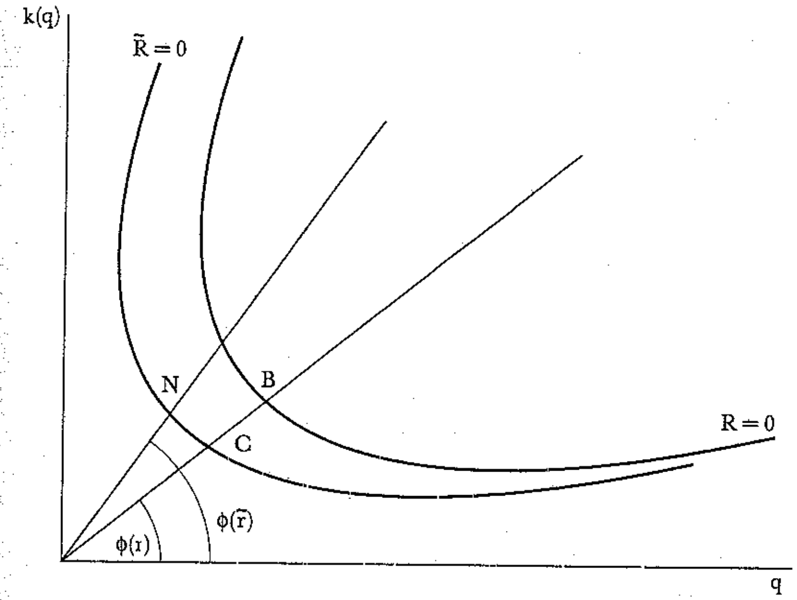

Figure 4.10 Margin displacement

PROPERTY 4.10 An increase in r/p causes an increase in the marginal quality of land and a decline in the capital intensity. An increase in c/p increases the marginal quality of land and does not affect the capital intensity.

This property is summarized in the first block of Table 4.1.

Total Agriculture

The size of cultivated land is determined by the marginal quality alone, and $\partial A/\partial z = -A(z) < 0$ or $E(A, z) = -zA(z)/A$. Combining this result with Property 4.10 we obtain the response function for cultivated land, $A(r/p, c/p)$. The total use of capital increases with $k(q)$, and unlike $k(q)$, it is also affected by z, $\partial K/\partial z = -K(z) < 0$. We combine this with Property 4.10 to obtain $K(r/p,\ c/p)$.[9] We can write an aggregate production function $Y = F(K, A)$, where Y is defined in (4.9). Using the results on the response of K and A, we summarize the supply function: $Y = F(r/p, c/p)$. These results are summarized in the first block of Table 4.1.

PROPERTY 4 11 Cultivated area, total capital use, and output all decline with r/p and c/p.

COROLLARY In this model, a change in the economic environment (r/p and c) affects land and capital in the same direction, and specifically, a land-augmenting shock is also land expanding

The difference in the responses of the two margins lies in their magnitude. To compare the strength of the response with changes in r, we use (4 18) and write $E(A, c) = E(A, z)$. Also, $E(A, r) = E(A, z)E(z, r)$, using (4 19)

$$\frac{E(k(q), r)}{E(A, r)} = -\frac{\sigma}{E(A, z)}\frac{S(c; z)}{S(k; z)}$$

The ratio is inversely related to the thickness of the land margin as given by $E(A, z)$ and to the share of capital in the output of the marginal land, $S(k; z)$. It is positively related to the demand elasticity of capital.

PROPERTY 4 12 When the demand elasticity of capital is "high," the change in capital, in response to a change in the rental rate, dominates that of land

This is an extremely important result. Under the choice of technique framework to be introduced in Chapter 6, the demand elasticity for capital in agriculture, in the domain of the coexistence of techniques, is likely to be very high, and consequently most of the increase in output comes from capital deepening.

If we turn to rent, the total rent to agriculture, R, is

$$R = \int_z^\infty R(q)A(q)dq = Y - rK - cA$$

Invoke the envelope theorem, $\partial R/\partial r = -K < 0$, $\partial R/\partial c = -A < 0$. We write the result in terms of elasticities:

$$\begin{aligned} E(R, r) &= -S(K)/S(R) \\ E(R, c) &= -S(c)/S(R), \end{aligned} \qquad (4\ 20)$$

where $S(K) = rK/Y$, and $S(c) = cA/Y$ respectively. As $S(K) + S(c) + S(R) = 1$, we can write

$$E(R, r) + E(R, c) = [S(R) - 1]/S(R)$$

It then follows

PROPERTY 4 13 The larger the share of total rent in total output, the more robust the total rent is to changes in the economic environment, represented here by r, c, and p.

Resource Constraint

In this model we replace the assumption that agriculture is a price taker in the market for capital goods with the assumption that capital is fixed at K, and consequently the economic problem is to allocate it to lands of different quality. The allocation decision is described by

$$\max_{k(q),z,r} L(k(q), z, r) = \int_z^\infty [qf(k(q)/q) - c]A(q)dq - r[\int_z^\infty A(q)k(q)dq - K], \qquad (4\ 21)$$

where r is the shadow price of capital, measured in units of output The FOC are equations (4.12) and (4 13) with p set at 1 and the resource constraint

$$L_3(\cdot) = \int_z^\infty A(q)k(q)dq - K = 0. \qquad (4.22)$$

The solution for $k(q)$ and z is similar to the first case. The difference is that now r is endogenous, and therefore we cannot solve for $k(q)$ independently from the solution for z and r With this qualification, the solution can be illustrated in terms of Figure 4.10

To obtain the response of resource allocation to the state variables K and c, we differentiate the first-order conditions. The derivative of the first of these conditions is given by equation (4 16) above, and that of the second is given by equation (4 17). Finally, to differentiate equation (4.22), we use equations (4.14) and (4 6) to rewrite it:

$$\Phi(r)Q = K. \qquad (4\ 23)$$

Hence $\Phi(r)dQ + \Phi'(r)Qdr = dK$ Use equation (4 6) to write $dQ = -zA(z)dz$ and write the differential of equation (4 23) as

$$-\Phi(r)zA(z)dz + \Phi'(r)Qdr = dK \qquad (4\ 24)$$

Use equation (4.14) to write equations (4.17) and (4.24) in matrix notations:

$$\begin{bmatrix} c & -\Phi(r)z^2 \\ -\Phi(r)zA(z) & \Phi'(r)Q \end{bmatrix} \begin{bmatrix} dz \\ dr \end{bmatrix} = \begin{bmatrix} zdc \\ dK \end{bmatrix} \tag{4.25}$$

The system is solved for dr and dz in terms of dK and dc. Label the matrix M, and note that except for M_{11}, all elements of M are negative; therefore det M is negative as well. As $M_{11} = c$, we obtain

$$c(\det M)^{-1} = \partial r/\partial K < 0. \tag{4.26}$$

Equation (4.26) bridges the two models, and the qualitative results of the first model are preserved.

The Effect of Capital

An increase in K causes a reduction of r which in turn increases the level of intensity and the rent on all cultivated lands. Consequently, the rent on the marginal land becomes positive, and the marginal quality declines.[10]

PROPERTY 4.14 An increase in capital causes a decline in the marginal quality land and in the real rental rate, r/p, and an increase in capital intensity. As a result, cultivated land, output, and total rent on land increase.

The strength of the response of A to K is proportional to the abundance of marginal land, measured by $E(A, z)$, to the capital intensity of the technology as measured by the share of capital on the marginal land, and to the response of r to changes in K as determined by the degree of concavity of the production function.

The elasticity $E(z, r)$ in this model has the same appearance as that of equation (4.19), but it is important to note that in this model both z and r are endogenous, and as such they vary only in response to changes in K or c. Thus, in the present case, the elasticity is just a characterization of the changes in z and r along the path generated by a change in K with c held constant.

Effect of Tax

The main difference of this model from the first one is that under a resource constraint a change in the tax affects the level of intensity. A rise in the tax rate causes a decline in the rent, and thereby the marginal quality land is forced out of production. This frees capital to be distributed to the cultivated land and thus causes an increase in the intensity of capital and a decline in the rate of return. This can be seen by solving equation (4.25) to obtain $\partial z/\partial c > 0$, $\partial r/\partial c < 0$, and

$$E(k(q), c) = E(k(q), r)E(r, c) > 0. \tag{4.27}$$

Qualitatively, the response of the size of the cultivated land is similar to that obtained in the first model:

$$E(A, c) = E(A, z)E(z, c) < 0. \tag{4.28}$$

When a unit of marginal land goes out of production, output declines by $zf(z)$ and increases by reallocation of capital to the cultivated land by $rk(z)$, the difference between these two terms is c, which is the net decline of output due to the increase in the tax. The total loss of output depends on the size of the margin. This result can be obtained formally by differentiating (4.11) with respect to z, using the optimal quantities of $k(q)$.

PROPERTY 4.15 An increase in the land real tax rate, c/p, causes an increase in the marginal quality land, a decline in the shadow rental rate, and a rise in capital intensity. As a result, cultivated land, output, and total rent decline.

We thus see that taxing land is not costless, and a decrease in the tax increases output. Any cost specific to land will have a similar effect to that of a land tax. It is important to differentiate between a fixed set-up cost and ongoing or variable cost. The first will affect the pace of reclamation of new land; the latter will affect the size of the land actually cultivated in a given year.

The qualitative results of this model are summarized in the second block of Table 4.1.

Output Quota

In the foregoing discussion, product demand was taken to be perfectly elastic. We now analyze the other extreme case, where the demand is perfectly inelastic, and examine the changes in factor demand caused by a change in output. The analysis is related to a variety of subjects, of which the most important is the differentiation between the substitution and expansion effects of a change in the economic environment on the demand for land. The substitution effect is relevant for the evaluation of the consequences of support programs with production quotas.

When output is held fixed, the problem of optimal resource allocation is that of cost minimization:

$$\min_{k(q),z,p} L(k(q), z, p) = rK + cA - p(Y - Y^*), \tag{4.29}$$

where A, K, and Y are defined in equations (4.5), (4.7), and (4.9) respectively. The Lagrange multiplier, p, is the marginal cost of producing Y^*. The interpretation is that when the demand is perfectly inelastic, the price is determined by the supply function. The FOC are

$$L_1 \equiv \partial L(\cdot)/\partial k(q) = r - pf'(q) = 0, \tag{4.30}$$

$$L_2 \equiv \partial L(\cdot)/\partial z = pzf[k(z)/z] - c - rk(z) = 0, \tag{4.31}$$

$$L_3 \equiv Y^* - \int_z^\infty qf(k(q)/q)A(q)dq = 0. \tag{4.32}$$

Differentiation of the FOC yields

$$dr = pf''(q)/q\, dk(q) + f'(q)\, dp, \tag{4.33}$$

$$c/z\, dz + zf(\cdot)\, dp = k(z)\, dr + dc,^{11} \tag{4.34}$$

$$dY^* - (m/p)\, dr = -mr/p^2\, dp - Y(z)\, dz,^{12} \tag{4.35}$$

where $m \equiv Q\Phi'(r/p)(r/p) < 0$. The system (4.34) and (4.35) can be written in matrix notation:

$$\begin{bmatrix} -mr/p^2 & -Y(z) \\ zf(z) & c/z \end{bmatrix} \begin{bmatrix} dp \\ dz \end{bmatrix} = \begin{bmatrix} 1 & -m/p & 0 \\ 0 & k(z) & 1 \end{bmatrix} \begin{bmatrix} dY^* \\ dr \\ dc \end{bmatrix}. \tag{4.36}$$

$-Y(z)$ is the only negative term in the matrices in (4.36). We can therefore sign the partial derivatives as follows:

$$\underset{- \; - \; +}{z(Y, r, c)}, \quad \underset{+ \; + \; +}{p(Y, r, c)}. \tag{4.37}$$

We solve for dp from equation (4.36) and use equation (4.33) to solve for $dk(q)$.

With these results we can now make inferences about the response of the rest of the system to a change in the economic environment. An increase in r causes an increase in p, but relatively less than the increase in r because capital is not the only input. Consequently, the ratio r/p rises, and $k(q)$ declines. To meet the output quota, land must expand, and therefore z must decline. The final outcome is also a decline in K, because of the increase in A while the level of output is unchanged. For z to decline in response to the rise in r, it is required that the rent on the marginal land increases; this is accomplished by the increase in p. If the rent increases on all quality land, the total rent to agriculture increases. This result reflects only the supply side, and it is unlikely that consumers would be willing to buy the initial quantity at higher prices. The actual implementation of such a scheme requires policy measures that would entail a welfare loss.

In contrast to the case of unrestricted output, summarized in Properties 4.11 and 4.12, under an output quota the demand for land and the demand for capital move in opposite directions. This movement represents the substitution effect, or a movement along an isoquant of the aggregate production function. The expansion effect is in the same direction for the two inputs. An increase in Y causes a decline in z, an increase in A, an increase in p, a decline in r/p, an increase in $k(q)$ for all q, and hence an increase in K. From this we infer that when output is unrestricted, the expansion effect, as in the case covered by Property 4.11, generated by a change in the factor prices, dominated the substitution effect. To obtain the point where the two effects on the demand for land are equal, we impose $dA = 0$ which implies $dz = 0$ and solve equation (4.36), allowing output to change. This is assigned as an exercise.

An increase in c causes an increase in z, a decline in A, and an increase in K. Also, p increases, hence r/p declines, and $k(q)$ increases for all q. The rent decreases due to the increase in the tax on agriculture. Here again, as in the case of capital constraint, a land tax affects resource allocation.

The results are summarized in Property 4.16 and in the third block of Table 4.1.

PROPERTY 4.16 Under an output quota, the quality of marginal land declines with r and increases with c. A change of either of these prices causes land and capital to move in opposite directions.

The Role of Prices and the Competitive Position of Agriculture

The foregoing analysis has repercussions for several attributes of cardinal importance for policy considerations. The nature and the strength of the response of agriculture to external or internal shocks is related to its competitive position, a concept we want to define explicitly.

DEFINITION The *competitive position* is measured by the factor share of land that makes up the agricultural-specific factor of production.

The higher the factor share of land, the less agriculture is susceptible to shocks. It is important to emphasize that the measure does not speak of the competitive position of farmers. Farmers who operate on the margin of their profitability because of low ability or because of past decisions affecting their balance sheets may be wiped out by unfavorable shocks. However, the land remains in agriculture and will be acquired by others Unfavorable shocks will thus cause revaluation of assets, sometimes severely, but the effect on output and resource utilization will be much milder

The effect of shocks on agriculture can then be viewed at two levels, the effect on land and the effect on output. The effect on land is related to the effect on the rent of the marginal land. When the rent becomes negative, land goes out of production. The effect on output is analyzed in Chapter 14

Exercises

4.1 (Food import)
A small open country is exporting nonagriculture (NA) and importing agriculture (A). Under what conditions, if any, would capital deepening (increasing k) increase jointly food consumption, production, and import?

4.2 (Equilibrium path)
Use the restricted demand function to show the case where capital accumulation causes the country to change from exporting to importing nonagriculture

4.3 (Equilibrium path under distortion)
Repeat the analysis of the equilibrium path of a closed economy generated by an increase in k for an economy that subsidizes agriculture. What is the similarity and the difference from that obtained for an economy free of intervention?

4.4 (Land economics)
Assume that the production function of quality q land is

$$Y(q) = qF[A(q), K(q)],$$

so that the quality does not affect the marginal rate of substitution of K and A Examine the three models discussed in Chapter 4. What propositions, if any, are affected by this change of specification of the production function?

4.5 Assume that the "land tax" is an increasing function of quality: $dc(q)/dq > 0$. Examine the case of agriculture as a price taker, using the production function in the chapter, and to simplify set $p = 1$ Derive the condition for the marginal quality.

4.6 Assume that output is exogenous
(a) There is a permanent decline in price (rental rate) of capital, r, whereas the tax on land, c, remains constant. Derive the change in the marginal cost and in the output under the condition that the extensive margin (z) is unchanged.
(b) Interpret the result.
Note: the point of departure is the equilibrium under an output quota